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ANALYSIS OF BRITTLE FRACTURE OF FOAM POLYSTYRENE PLATES WITH HOLES

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A series of experiments was conducted on brittle fracture of foam polystyrene plates in uniform uniaxial tension and in the presence of stress concentration near slits and holes. Experimental data were analyzed using a numerical algorithm because the ratio between the dimensions of the holes and those of the plates differed from that in known problems for infinite planes and strips with holes. The experimental and calculated data are compared.

Numerical Algorithm. A numerical algorithm for strength analysis of plane structural members with stress concentrators was developed in [1]. This algorithm uses the gradient fracture criterion and the boundary-element method (the method of fictitious loads). In the present paper, this algorithm is applied to analysis of plates of finite dimensions.

The essence of the gradient approach to evaluation of fracture is that the nonuniformity of a stressed state leads to reduction in the breaking ability of stress in the range of maximum values. According to the two-parameter gradient strength criterion formulated in [2, 3], in order to determine the breaking load, the ultimate strength of the material $\sigma_{\rm t}$ should be compared not to the first principal stress σ_1 (used as the equivalent stress) but to a certain effective stress $\sigma_{\rm eff} = \sigma_1/f(g_1, L_1, \beta) < \sigma_1$. The denominator $f(g_1, L_1, \beta)$ is a function of the stress-field nonuniformity at a point considered and of two parameters that depend on the material properties. The stress-state nonuniformity is characterized by the relative gradient of the first principal stress $g_1 = |\operatorname{grad} \sigma_1| / \sigma_1$ and is determined from the elastic solution of the corresponding problem. The parameter L_1 has the dimension of length and is determined from the condition of agreement between the gradient criterion and linear fracture mechanics: $L_1 = (2/\pi) K_{\rm Ic}^2/\sigma_{\rm t}^2$, where $K_{\rm Ic}$ is the critical coefficient of stress intensity. The nondimensional parameter β, which ranges from 0 to 1, allows for the quasibrittle nature of fracture: $\beta = \sigma_{\rm t}/(E\varepsilon_*)$, where ε_* is the strain at the moment of fracture in uniform uniaxial tension and E is the elasticity modulus.

The main distinction of the numerical algorithm developed in [1] is that in calculations, not only the stressed-state components are determined (by the boundary-element method) but also their derivatives with respect to spatial coordinates.

Experimental Results. Experiments on brittle fracture were performed with foam polystyrene plates of trade-mark PSB-15 with dimensions of 1 × 1 m and a thickness of 10 cm.

The ultimate strength of the material was determined in tension tests for plates consisting of two specimens joined in parallel (Fig. 1). Such geometry reduced bending stresses in the specimens and ensured that failure occurred in the working section but not in the fillets. It should be noted that the strain diagram for polystyrene in the stress–strain coordinates was linear up to the moment of failure. The failure strain ε_* reached approximately 1% (the average value over three experiments was 1.023%), and the average value of the limit stress was $\sigma_{\rm t} = 78.415$ kPa, which agrees with the requirements of the State Standard [4]. In tests of plates with cracks whose total length was 0.2 of the plate width, the critical stress-intensity coefficient was $K_{\rm Ic} = 16.046$ kPa $\cdot$ m$^{1/2}$ (averaged over three experiments and obtained taking into account the finite width of the plates and using the formula proposed by Feddersen [5]). With the values of $\sigma_{\rm t}$ and $K_{\rm Ic}$ known, the parameter L_1 in the gradient fracture criterion was found:

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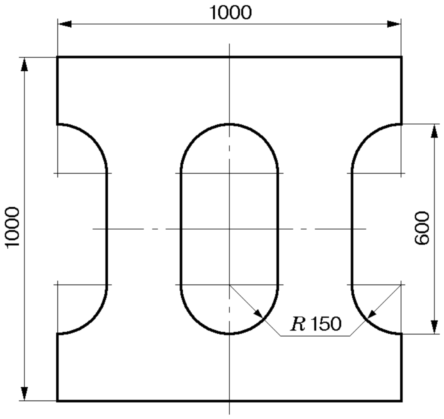

Fig. 1

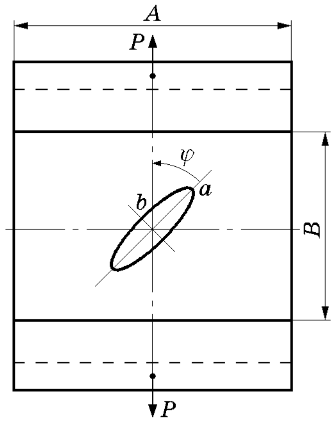

Fig. 2

TABLE 1

Geometric Parameters of the Specimens

Test No.	A, mm	B, mm	Shape of the hole	a, mm	b, mm	φ, deg
1	990	690	Circle	200	200	—
2	1004	700	Circle	202	202	—
3	990	690	Ellipse	210	65	90
4	991	696	Ellipse	202	51	90
5	990	693	Ellipse	202	51	45
6	995	700	Ellipse	202	50	45

TABLE 2

Critical Average Stress σ_* [Pa]

Test No.	Experimental data	Gradient criterion		Classical criterion	
		Model I	Model II	Model I	Model II
1	43,532	15,164 (−65.2)	28,194 (−35.2)	12,955 (−70.2)	24,460 (−43.8)
2	31,565	15,228 (−51.8)	28,193 (−10.7)	13,031 (−58.7)	24,493 (−22.4)
3	17,949	12,393 (−31.0)	18,632 (3.8)	6552 (−63.5)	9912 (−44.8)
4	16,981	12,634 (−25.6)	18,116 (6.7)	5782 (−66.0)	8341 (−50.9)
5	28,682	18,211 (−36.5)	27,002 (−5.9)	9646 (−66.4)	16,128 (−43.8)
6	26,803	18,309 (−31.7)	26,884 (0.3)	9619 (−64.1)	15,968 (−40.4)

Note. The numbers in parentheses are the deviations (in percent) of the calculated values from the experimental results.

$L_1 = 2.6657$ cm. The parameter β in that criterion was set equal to unity since brittle fracture of the material was observed.

A series of experiments on fracture of plates with central holes of circular or elliptic form (Fig. 2) was conducted. The geometric parameters of the specimens are listed in Table 1 (a and b are the semiaxes of the hole). The major axis of an elliptic hole was either perpendicular to the direction of stretching or at an angle $\varphi = 45°$ to that direction. The breaking force P_* was measured by a dynamometer.

Comparison of Numerical Estimates with Experimental Data. Load was applied to the foam polystyrene plates along the vertical by means of clamps which were made of plywood sheets and in which the upper and lower portions of the plate were fixed (Fig. 2). The plywood sheets were bolted together by stud bolts and clamped the edges of the plates. The distance B between the upper and lower clamps (working length of a specimen) was approximately 70 cm. Such fixing and loading conditions were modeled by constant (over the plate width) load (model I) or by constant (over the plate width) vertical displacement (model II). For calculations using model II, Young's modulus ($E = 7.666$ MPa) and Poisson's ratio ($\nu = 0.36$) were determined experimentally. The total number of boundary elements in the problem was 300, of which 100 elements were assigned to the hole and 50 elements to each of the four sides of the plate. Experimental results and numerical estimates are listed in Table 2.

In the experiments, critical average stress σ_* was calculated as the ratio of P_* to the width and thickness of the plate in a cross section without a hole. In numerical calculations using model I, the critical average stress σ_* was determined as the ratio $\sigma_{\rm t}/\sigma_{\rm eff}$ under unit load, and for model II, it was obtained by averaging the normal stresses over the plate width at the moment the effective stress reaches the ultimate strength at the most dangerous point on the hole contour. The numerical estimates of the breaking load predicted by model I are smaller than the experimental values. The critical loads predicted by model II are higher than those in model I and approach experimental values, especially the values for plates with elliptic holes obtained using the gradient criterion.

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